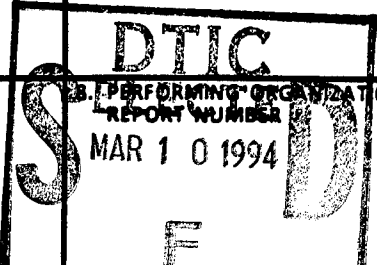


REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
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1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE Jan. 24, 1995		3. REPORT TYPE AND DATES COVERED Final (April 1, 1993 - November 30, 1994)
4. TITLE AND SUBTITLE  Wavelet Analysis and Its Applications			5. FUNDING NUMBERS  DAAH 04-93-G-0047	
6. AUTHOR(S)  Charles K. Chui				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Charles K. Chui Department of Mathematics Texas A&M University College Station, TX 77843-3368				
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSORING/MONITORING AGENCY REPORT NUMBER  ARO 31003.1-MA-SDI	
11. SUPPLEMENTARY NOTES The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  Abstract. Time-frequency localization is one of the most essential features of the wavelet transform. It was shown that while high order Daubechies and Battle-Lemarie wavelets give poor time-frequency localizations, the Chui-Wang spline-wavelets provide asymptotically optimal time-frequency windows. On the other hand, we also showed that by using the scale 3 instead of 2, symmetry can be achieved by orthonormal wavelets with compact support. Multivariate wavelets, particularly those with matrix dilation, were studied, and the theory of oversampling frames was extended to this setting. Interpolating wavelets have distributional duals that lead to the notion of functional wavelet transform. Other extensions required a study of the stability issue and algorithmic construction in multivariate splines. Applications to systems theory lead to the study of Hankel approximation and localization of neural networks.				
14. SUBJECT TERMS Time-frequency localization, symmetry for compactly supported orthonormal wavelets, multivariate wavelets with matrix dilations, interpolating wavelets and functional wavelet transform, neural networks localized by splines and wavelets.			15. NUMBER OF PAGES	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED		18. SECURITY CLASSIFICATION UNCLASSIFIED		19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED
			20. LIMITATION OF ABSTRACT UL	

DTIC QUALITY INSPECTED 2

19950308 185

# Wavelet Analysis and Its Applications

## FINAL REPORT

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January 24, 1995

Sponsored by  
SDIO/IST

U.S. Army Research Office

Contract Number DAAH 04-93-G-0047

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## **A. Statement of the problem studied**

This research project was devoted to the study of the mathematical aspects of wavelet analysis and to investigate the various applications of wavelets and related areas. The original research plan was a three-year effort to fully develop the stationary theory of wavelets, both in the univariate and the multivariate settings, and to apply this theory to solve certain important engineering problems, including such applications as modeling the cochlea of the human ear. Unfortunately, due to budget difficulty, this research project was only funded for the first year. Although an additional fund greatly facilitate the six-month extension, there was not enough time nor manpower to complete the original somewhat ambitious goal. Nevertheless, a total of fourteen (14) papers were written and published (or accepted), and further research is being carried out, even without research support.

## **B. Summary of the most important results**

The core of this research effort is the mathematical development of wavelet analysis. Hence, although the results obtained can be divided in the following three categories, the main results are in wavelet theory.

### **B.1. Wavelet Analysis**

A time-frequency analysis of the wavelet transform was studied very carefully in [1]. It was shown that families of semi-orthogonal wavelets (including the Chui-Wang spline wavelets) provide asymptotically optimal time-frequency localization [1], while the width of the time-frequency window of orthogonal wavelets (including the Daubechies and Battle-Lemarie wavelets) tends to infinity as the order increases. For applications to signal analysis, the complexity of the semi-orthogonal (particularly the Chui-Wang spline) wavelet decomposition and reconstruction schemes is given in [3]. Since orthonormal wavelets are commonly used, particularly by the engineers, we changed the scale from 2 to 3 in our paper [4] in order to yield compactly supported orthonormal wavelets with symmetry and anti-symmetry. It is important to remark that this requires the sacrifice of minimum support. To generalize to the multivariate setting, a general framework of multivariate wavelets together with their duals was developed in [5]. In particular, when a matrix dilation is used, a general theory which includes that of oversampling frames was given in [6]. All of the wavelet theory mentioned above is a result of orthogonal projection. In [7], we considered wavelets obtained by interpolation projection and introduced the concept of functional wavelet transform induced by the distributional duals. A new method of construction of spline-wavelets was introduced in [8].

### **B.2. Spline modeling**

Due to the fact that spline-wavelets are simple, explicit, and flexible, and that near-optimality for time-frequency localization is achieved, we returned to further develop the algorithmic and stability aspects of multivariate, and particularly bivariate, spline functions on arbitrary triangulations. When vertex splines are used to achieve the optimal order of approximation from the spline spaces of degree  $3r + 2$  with  $r$ th order of smoothness, it

is well-known that instability occurs as a result of near-singularity at a vertex of degree 4. In our paper [9], the supports of vertex splines are enlarged, to include an interior edge if necessary, to assure stability. In order to lower the degree from  $3r + 2$  to  $3r + 1$  while maintaining the optimal order of approximation, it is necessary (even for  $r = 1$  and the regular three-directional mesh) to refine the triangulations. In [10], we focussed on the case  $r = 1$  but allowing an arbitrary triangulation, and developed an algorithm for some minimum refinement.

### B.3. Systems theory

Recently, there has been some interest in the investigation of the relevance of wavelets in the theory of linear systems. In an attempt to extend the notion of the continuous (or integral) wavelet transform (CWT) to the half-plane in our work [11], we related this generalized CWT to the Hankel transform in systems analysis. In [12], we proved that the Hankel approximation is continuous even when the  $s$ -numbers (or singular values) are not simple, and hence, assured the convergence of the corresponding rational approximants. A comprehensive study of rational approximation in system engineering was given in [13]. For neural systems, we studied networks with one hidden layer in [14] and incorporated splines and spline-wavelets for local approximation and analysis.

### C. List of publications

1. C. K. Chui and J. Z. Wang, A study of compactly supported scaling functions and wavelets, in *Curves and Surfaces*, P. J. Laurent, A. LeMéhauté, and L. L. Schumaker (eds.), A. K. Peters Publ., Wellesley, Mass. 1994, 121–140.
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5. C. K. Chui and C. Li, A general framework of multivariate wavelets with duals, *Appl. and Comp. Harmonic Anal.* **1** (4) (1994), 368–390.
6. C. K. Chui and X. L. Shi, Affine operators and frames of multivariate wavelets, in *Advances in Computational Mathematics*, H. P. Dikshit and C. A. Micchelli (eds.), World Scientific Publ., 1994, 139–155.
7. C. K. Chui and C. Li, Dyadic affine decompositions and functional wavelet transforms, accepted for publication in *SIAM Math. Analysis*.
8. C. K. Chui and J. M. de Villiers, Applications of optimally local interpolation to constructing interpolatory approximants and compactly supported wavelets, accepted for publication in *Math. Comp.*
9. C. K. Chui, D. Hong, and R. Q. Jia, Stability of optimal order approximation by bivariate splines over arbitrary triangulations, accepted for publication in *Trans. American Math. Soc.*

10. C. K. Chui and D. Hong, Construction of local  $C^1$  quartic spline elements for optimal order approximation, submitted to *Math. Comp.*
11. C. K. Chui and X. Li, Generalized wavelet decompositions of bivariate functions, *Proc. of Amer. Math. Soc.* **121** (10) (1994), 125–131.
12. C. K. and X. Li, Continuity of best Hankel approximation and convergence of near-best approximants, *SIAM J. Control & Optimization* **32** (6) (1994), 1769–1781.
13. C. K. Chui and G. Chen, Rational approximation theory and its applications in system engineering: An overview, *Proc. 33rd Conf. on Decision and Control*, Lake Buena Vista, FL, December 1994, 1487–1492.
14. C. K. Chui, X. Li, and H. N. Mhaskar, Neural networks for localized approximation, *Math. Comp.* **63** (1994), 607–623.

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 Jian-ao Lian (Ph. D. granted, Aug. 1993)